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Overvalued and Undervalued Exchange Rates in an Equilibrium Optimizing Model

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For some problems it helps to think of overvalued or undervalued exchange rates as the result of unsustainable macroeconomic policies — such as undertaxation or overspending in the public sector — rather than the result of markets failing to clear or economic agents failing to behave in an optimizing manner.

Intertemporal equilibrium optimizing models have recently become the standard framework for analyzing such macroeconomic issues as terms of trade, fiscal or trade policy, international transfers, supply shocks, and technological progress.

They have rarely been used to discuss overvalued and undervalued exchange rates — probably partly because an equilibrium model is not usually considered appropriate for examining a “disequilibrium” situation.

For some kinds of problems, however, it may be more reasonable to think of overvalued or undervalued exchange rates as the result of unsustainable macroeconomic policies rather than the result of markets failing to clear or economic agents failing to behave in an optimizing manner. It may be useful to examine the issue of undervalued or overvalued exchange rates in a framework that forces us explicitly to take into account the economic agents’ maximizing behavior and budget constraints over time.

In the model Lizondo presents, sustainable fiscal policies produce an equilibrium real exchange rates, and unsustainable policies produce misaligned exchange rates. When the exchange rate is overvalued, maintaining present fiscal policies means the present value of lifetime public sector spending would be higher

than the present value of taxes.

Misaligned exchange rates imply both intertemporal and intersectoral shifts in the economy’s pattern of expenditure. An overvalued exchange rate, for example, implies that an increase in present expenditures must be balanced by a reduction in future expenditures — reflected in a worsening of the current account.

Whether the expenditure shifts from the public to the private sector or the reverse depends on how the misalignment was brought about and is to be compensated for. If it was brought about by increased public sector spending that is to be compensated for by higher future taxes, the shift will be from the private to the public sector. If it was brought about by lower taxes that are to be compensated by lower future public sector spending, the shift will be from the public to the private sector.

For Lizondo’s model to be used to obtain welfare conclusions about the use of overvalued or undervalued exchange rates, it would be necessary to assign some social value to public sector expenditure. Other fruitful modifications of the model would be to incorporate investment activity as well as money — the latter of which would allow for discussion of the effects of monetary policy, exchange rate arrangements, and nominal exchange rate policy.

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I. Introduction

The purpose of this paper is to discuss the concepts of overvalued and undervalued exchange rates in the context of an intertemporal equilibrium optimizing model. This is done by linking those concepts to the idea of sustainable and unsustainable macroeconomic policies.

The appropriate level of the real exchange rate has always been an important issue in policy discussions regarding macroeconomic performance, particularly in developing countries. In these discussions, the concepts of overvalued and undervalued exchange rates are frequently used to refer to situations in which the real exchange rate is considered to be "too high" or "too low" respectively, in relation to its "correct" or "equilibrium" level.¹ The term "misaligned" exchange rate is generally used to denote any of these two type of situations.

Despite the frequent use of these expressions in the literature, there is not a unique definition of overvalued and undervalued exchange rates. Sometimes, these terms are used to refer to short term phenomena. Thus, for example, in models with sticky prices the real exchange rate may take some time to adjust to changes in exogenous or policy variables, both in economies with predetermined nominal exchange rates and in economies with flexible nominal exchange rates. In the case of predetermined exchange rates, a devaluation would increase the prices of traded goods immediately, but would affect the prices of nontraded goods only with some lag, thus producing transitory deviations in the real exchange rate from its long run level. In the case of flexible exchange rates, changes in the nominal quantity of money would produce an immediate reaction in the nominal exchange rate but only a lagged response in the prices of domestic goods, thus resulting also in transitory deviations of the real exchange rate from its long run level. Short term deviations of this

¹We define the real exchange rate as the relative price of nontraded with respect to traded goods. Therefore, an increase in the real exchange rate implies a real appreciation, while a decline in the real exchange rate implies a real depreciation.

type are not necessarily related to sticky prices, they may also be the result of some other transitory events such as speculative bubbles that drive the exchange rate away from its long run equilibrium level.² In all these cases, the deviations of the real exchange rate from its long run level can be interpreted as overvaluations or undervaluations, and they all apply to short term phenomena.

In some other cases the notions of overvalued and undervalued exchange rates refer to longer term situations. For example, it is sometimes discussed whether it would be beneficial for developing countries to implement policies leading to a sustained undervaluation of their domestic currencies.³ Furthermore, these concepts have also been used to examine issues associated with economic growth.⁴ Clearly, definitions of misaligned exchange rates derived from models based on short term price stickiness or speculative activities are not the most relevant for the discussion of some of these long term issues.

This paper presents an interpretation of the notions of overvalued and undervalued exchange rates that may be more useful for the analysis of long term situations. In this interpretation, misaligned exchange rates are associated with macroeconomic policies that are unsustainable, given the set of restrictions that the economy faces, such as terms of trade, world interest rates, level of foreign indebtedness, etc. The misalignment of the exchange rate is the deviation of the actual level of the real exchange rate with respect to the level that it would have attained under sustainable policies. The discussion is based on an intertemporal equilibrium optimizing model with uncertainty.

Intertemporal equilibrium optimizing models have become recently the standard framework for analyzing a variety of open economy macroeconomic issues. Those issues include, among others, the effects of changes in the terms of trade, fiscal policies, trade policy,

²See for example Dornbusch (1986).

³See for example Dornbusch (1988), and Fischer (1988).

⁴See Rodrik (1986).

international transfers, supply shocks, and technological progress.⁵ This framework, however, has rarely been used to discuss overvalued and undervalued exchange rates.⁶ Probably, this is due in part to the conviction that an equilibrium model cannot be used to examine what is considered to be a "disequilibrium" situation. However, for some type of problems, it may be more reasonable to think of an overvalued or undervalued real exchange rate as the result of unsustainable macroeconomic policies rather than the result of markets failing to clear, or economic agents failing to behave in an optimizing manner. Thus, it is useful to examine this issue within a framework that has proved to be very helpful in discussing other questions, and which forces us to take into account explicitly the economic agents' maximizing behavior and intertemporal budget constraints.

The rest of the paper is organized as follows. Section II presents an intertemporal equilibrium optimizing model and defines

⁵ See for example Brock (1988), Bruno (1982), Devereux (1987), Djajic (1987), Dornbusch (1983), Edwards (1989), Frenkel and Razin (1988), Greenwood (1984), Marion (1984), Murphy (1986), and Razin (1984).

⁶ An exception is Rodrik (1986). In his equilibrium model, he defines overvaluation for a fixed nominal exchange rate system as the deviation of the actual real exchange rate with respect to the real exchange rate that would prevail under flexible nominal rates, caused by a forward shift in time of government expenditure. Edwards (1989), also has a chapter devoted to an intertemporal equilibrium model. The real exchange rate that results from solving that model is defined as the equilibrium real exchange rate, and misalignments are defined as departures from that rate. However, under those definitions his model cannot be used to examine misaligned exchange rates, because that model can only generate equilibrium levels of the real exchange rate. Thus, in the same book, a subsequent chapter which discusses overvaluation, uses an entirely different model in which behavioral relationships are directly postulated rather than being derived from an optimizing equilibrium framework. Although in some aspects the models in Rodrik (1986) and Edwards (1989) are more general than the model presented in this paper, they do not incorporate uncertainty.

sustainable and unsustainable policies.⁷ Section III solves the model for the case of sustainable policies. Section IV examines the effects of departures from sustainable policies. Finally, section V establishes a relationship between the concepts of sustainable and unsustainable policies, and the idea of overvalued and undervalued exchange rates.

II. The Model

Assume a small open economy that exists for two periods indexed by i , with $i=1$ indicating the present, and $i=2$ the future. Both, the private and the public sector can borrow and lend freely at the fixed world interest rate r expressed in terms of traded goods. The private sector produces traded and nontraded goods along a concave transformation curve, and also consumes both types of goods. Production and private sector consumption of traded and nontraded goods in period i are denoted by x_{ti} , x_{ni} , c_{ti} , and c_{ni} , respectively. The public sector consumes traded goods g_{ti} , and nontraded goods g_{ni} , and collects taxes t_i .

It will be convenient to discuss first the public sector behavior. It is assumed that out of total public sector expenditure g_i in terms of traded goods, a fraction λ is devoted to traded goods and a fraction $(1-\lambda)$ to nontraded goods. Thus,

$$(1) \quad g_{ti} = \lambda g_i$$

$$(2) \quad g_{ni} = (1-\lambda) q_i^{-1} g_i$$

where q_i is the relative price of nontraded goods in terms of traded goods, which we will refer to as the real exchange rate. Taxes t_i are determined in terms of traded goods. The public sector must respect its intertemporal budget constraint. Assuming there are no public

⁷ This is a model without money, so sustainable and unsustainable policies refer to fiscal policies.

sector's assets or debt at the beginning of period one⁸:

$$(3) \quad (g_1 - t_1) = (1+r)^{-1}(t_2 - g_2)$$

Thus, a present deficit must be financed by a future surplus, and viceversa.

The public sector's budget constraint is the basis for our definition of "sustainable" policies. We define a sustainable policy in period 1, as a combination of t_1 and g_1 that can also be maintained during period 2 while respecting the public sector's intertemporal budget constraint. Therefore, a sustainable policy requires $t_1 - g_1$, which can be repeated in period 2 without violating equation (3).⁹ In contrast, any policy that does not result in budget equilibrium in period 1 is unsustainable, since taxes and/or expenditure will have to be adjusted in the future to comply with the intertemporal budget constraint.

We will use as an initial situation one with sustainable policies $t_1 - g_1 = 0$, and we will discuss departures from this situation. We assume that if policies in period 1 are sustainable they will be repeated in period 2. Thus, $t_2 - g_2 = 0$. However, if policies in period 1 are unsustainable, either taxes or public sector's expenditure will be adjusted in period 2. We assume that it is uncertain as of period 1 which of the components of the budget will be adjusted in period 2. There are two alternatives:

$$(4) \quad g_2 = 0 \quad t_2 = 0 + (1+r)(g_1 - t_1) \quad \text{with probability } p, \text{ and}$$

$$(5) \quad t_2 = 0 \quad g_2 = 0 + (1+r)(t_1 - g_1) \quad \text{with probability } (1-p).$$

Thus, the adjustment in period 2 takes place via taxes with

⁸ This simplifies the presentation without affecting the conclusions.

⁹ If the public sector has a debt of d at the beginning of period one, a sustainable policy requires $t_1 - g_1 + d(1+r)(2+r)^{-1}$. As mentioned above, the treatment of this more general case would just make the presentation more cumbersome without affecting the conclusions.

probability p , and via public sector's expenditure with probability $(1-p)$.

The private sector is assumed to derive utility from the consumption of traded and nontraded goods in both periods. Lifetime utility is additively separable, with each period's utility defined over an aggregate consumption index c_i , which has a Cobb-Douglas form, and with the elasticity of intertemporal substitution in consumption equal to one. Thus,

$$(6) \quad W = U(c_1) + (1+\rho)^{-1} U(c_2) \quad \text{where} \quad U(c_i) = \ln c_i$$

$$\text{and} \quad c_i = c_{ti}^\alpha c_{ni}^{1-\alpha}$$

where ρ is the private sector's rate of time preference. Since future fiscal policy is uncertain - unless the present budget is in equilibrium - the private sector is assumed to maximize expected utility in period one. Thus, the private sector chooses x_{t1} , x_{n1} , c_{t1} , and c_{n1} , so as to maximize

$$(7) \quad E W = U(c_1) + (1+\rho)^{-1} E U(c_2)$$

subject to

$$(8) \quad x_{ni} = F(x_{ti}) \quad \text{with } F' < 0 \text{ and } F'' < 0$$

$$(9) \quad (c_{t1} + c_{n1}q_1) + (1+r)^{-1}(c_{t2} + c_{n2}q_2) + t_1 + (1+r)^{-1}t_2 =$$

$$(x_{t1} + x_{n1}q_1) + (1+r)^{-1}(x_{t2} + x_{n2}q_2)$$

$$(10) \quad x_{ni} = c_{ni} + g_{ni} = c_{ni} + (1-\lambda)q_i^{-1}g_i$$

Equation (8) indicates that production takes place along a concave transformation curve. Equation (9) is the private sector intertemporal budget constraint indicating that the present value of private sector consumption plus tax payments must be equal to the present value of

output¹⁰. Equation (10) is the equilibrium condition for the nontraded goods market, equating supply to private sector plus public sector demand.

III. Sustainable Policies

We will derive first the solution under a sustainable policy in order to illustrate some properties of this solution. Under a sustainable policy $t_1 = g_1 = t_2 = g_2 = \theta$, so there is no uncertainty. The private sector maximizes (6) subject to (8)-(10). Using the public sector's intertemporal budget constraint to replace for tax payments in (9), this problem can be written as maximizing:

$$(11) \quad W = \alpha \ln c_{t1} + (1-\alpha) \ln [F(x_{t1}) - (1-\lambda)q_1^{-1}\theta] + \\ + (1+\rho)^{-1} (\alpha \ln c_{t2} + (1-\alpha) \ln [F(x_{t2}) - (1-\lambda)q_2^{-1}\theta])$$

with respect to c_{t1} , c_{t2} , x_{t1} and x_{t2} , subject to

$$(12) \quad c_{t1} + (1+r)^{-1}c_{t2} = x_{t1} + (1+r)^{-1}x_{t2} - \lambda \theta [1+(1+r)^{-1}]$$

The first order conditions for a maximum are

$$(13) \quad \alpha c_{t1}^{-1} = \delta$$

$$(14) \quad (1-\alpha) [-F'(x_{t1})][F(x_{t1}) - (1-\lambda)q_1^{-1}\theta]^{-1} = \delta$$

$$(15) \quad \alpha c_{t2}^{-1} = (1+\rho)(1+r)^{-1} \delta$$

$$(16) \quad (1-\alpha) [-F'(x_{t2})][F(x_{t2}) - (1-\lambda)q_2^{-1}\theta]^{-1} = (1+\rho)(1+r)^{-1} \delta$$

and constraint (12), where δ is the value of the Lagrangean multiplier associated with that constraint. Note also that in equilibrium q_1^{-1} must be equal to the slope of the transformation

¹⁰ It is assumed that the private sector holds no assets or debts at the beginning of period one.

curve, $[-F'(x_{t1})]$.

The solution to this maximization problem depends on the relationship between r and ρ . For example, take the case of $r=\rho$. It is clear from (13)-(16) that the solution implies $c_{t1}=c_{t2}$, $x_{t1}=x_{t2}$, and $q_1=q_2$. Thus, in this case the second period is an exact copy of the first one, and the current account is in equilibrium in both periods. Now, assume that $r < \rho$. Equations (13)-(15) imply that $c_{t1} > c_{t2}$, $x_{t1} < x_{t2}$, $q_1 > q_2$, and there is a current account deficit in the first period compensated by a current account surplus in the second period. The reason is the following. Since the private sector's rate of time preference is higher than the rate of interest, the private sector prefers to borrow in order to consume more of both goods in the present and pay back in the future. A higher consumption of nontraded goods implies a higher production and a higher relative price of those goods, i.e. a higher real exchange rate. A higher production of nontraded goods, in turn implies a lower production of traded goods, which together with a higher consumption, imply a current account deficit. All these changes are compensated by changes in the opposite direction in the second period. Thus, in the second period there is a lower consumption of both types of goods, a higher production of traded goods, a current account surplus, and a lower real exchange rate. It is clear therefore that the solution under sustainable policies does not necessarily imply a flat profile for production, consumption, the real exchange rate, or the current account balance.

IV. Departures from Sustainable Policies

We will use now the solution under sustainable policies as the benchmark for assessing the effects of the adoption of unsustainable policies in our model. When the public sector's budget is not in equilibrium in the first period, future fiscal policy must be different from the present one. Either taxes or expenditure must be adjusted so as to comply with the intertemporal budget constraint. In this case, all the variables dated $i=2$ in equations (7)-(10) are uncertain at the time the private sector makes production and consumption decisions for period 1. Therefore, the private sector must solve this stochastic dynamic programming exercise by first

maximizing utility in period 2 conditional on any arbitrary decision for period 1, and the alternative fiscal policies for period 2. Then it must take expected value of utility in period 2 over the alternative fiscal policies for that period. And finally, it must choose the level of the decision variables in period 1 so as to maximize (7). We will follow those steps.

Conditional maximization of utility in period 2

Using equations (1), (2), (6), (8), and (10), at time 2 the private sector maximizes

$$(17) \quad \alpha \ln c_{t2} + (1-\alpha) \ln [F(x_{t2}) - (1-\lambda)q_2^{-1}g_2]$$

with respect to c_{t2} and x_{t2} , subject to

$$(18) \quad c_{t2} = x_{t2} + (1+r)S_1 + (1-\lambda)g_2 - t_2$$

where S_1 is private sector savings in the first period, and is defined by

$$(19) \quad S_1 = x_{t1} - c_{t1} + (1-\lambda)g_1 - t_1$$

Using the public sector intertemporal budget constraint (3), and equation (19), constraint (18) becomes

$$(20) \quad c_{t2} = x_{t2} + (1+r)(x_{t1} - c_{t1} - \lambda g_1) - \lambda g_2$$

The first order conditions for a maximum are

$$(21) \quad \alpha c_{t2}^{-1} = \delta$$

$$(22) \quad (1-\alpha)[-F'(x_{t2})] \{F(x_{t2}) - (1-\lambda)[-F'(x_{t2})]g_2\}^{-1} = \delta$$

and constraint (20), where δ is the value of the Lagrangean multiplier associated with that constraint.

Budget adjustment via taxes in period 2

As indicated by equations (4) and (5), there are two alternatives for fiscal policy in period 2. The first alternative, shown in equation (4), is $g_2 = \theta$; the adjustment in the second period takes place via taxes. In this case, and after eliminating δ , the first order conditions become

$$(23) \quad \alpha (F(x_{t2}) - (1-\lambda)[-F'(x_{t2})]\theta) - (1-\alpha) c_{t2} [-F'(x_{t2})]$$

$$(24) \quad c_{t2} = x_{t2} + (1+r)(x_{t1} - c_{t1}) - \lambda\theta - \lambda(1+r)g_1$$

Choosing the optimum values for x_{t2} and c_{t2} , we obtain a function that indicates the utility in period 2 conditional on fiscal policy $g_2 = \theta$, and arbitrary levels of the decision variables in the first period.

$$(25) \quad \Omega^*(x_{t1}, c_{t1}, g_1) \quad \text{with} \quad \Omega_{x_{t1}}^* = (1+r)\delta^* \quad \text{and} \quad \Omega_{c_{t1}}^* = -(1+r)\delta^*$$

where δ^* is the value of the multiplier at the optimum.

In order to see how the optimum values of x_{t2} and c_{t2} change with changes in the arguments of function Ω^* , we obtain from the first order conditions:

$$(26) \quad \begin{bmatrix} A_2 & 1 \\ -x_{t2} & c_{t2} \end{bmatrix} \begin{bmatrix} \hat{x}_{t2} \\ \hat{c}_{t2} \end{bmatrix} = \begin{bmatrix} 0 \\ (1+r)(x_{t1}\hat{x}_{t1} - c_{t1}\hat{c}_{t1}) - \lambda\theta(1+r)\hat{g}_1 \end{bmatrix}$$

where a symbol $\hat{}$ over a variable denotes percentage change

$$\beta_1 = g_{n1}/c_{n1} > 0$$

$$A_1 = -(1+\beta_1)(\eta_{q_1 x_{t1}} + \eta_{x_{n1} x_{t1}}) > 0$$

$\eta_{q_1 x_{t1}} < 0$, is the elasticity of the real exchange rate with respect to the production of traded goods,

$\eta_{x_{t1}x_{t1}} < 0$, is the elasticity of the production of nontraded goods with respect to the production of traded goods. Both elasticities are evaluated along the transformation curve.

The determinant of the system in (26) is

$$(27) \quad \Delta_1 - A_2 c_{t2} + x_{t2} > 0$$

and the changes in the values of the optimum choices for production and consumption of traded goods in the second period are

$$(28) \quad \hat{x}_{t2} = \Delta_1^{-1} [(1+r)(c_{t1}\hat{c}_{t1} - x_{t1}\hat{x}_{t1}) + \lambda\theta(1+r)\hat{g}_1]$$

$$(29) \quad \hat{c}_{t2} = \Delta_1^{-1} A_2 [(1+r)(x_{t1}\hat{x}_{t1} - c_{t1}\hat{c}_{t1}) - \lambda\theta(1+r)\hat{g}_1]$$

We also know from the first order condition (21) that

$$(30) \quad \hat{\delta}^* = -\hat{c}_{t2}$$

Budget adjustment via public sector's expenditure in period 2

Equations (25)-(30) provide us with all the necessary information for knowing the private sector choices in period 2 under any arbitrary decision in period 1, when the adjustment in the second period takes place via taxes. The same analysis must be carried out for the case in which the adjustment takes place via public sector's expenditure, $g_2 = \theta + (1+r)(t_1 - g_1)$. In this case, the first order conditions become

$$(31) \quad \alpha \{ F(x_{t2}) - (1-\lambda)[-F'(x_{t2})][\theta + (1+r)(t_1 - g_1)] \} = (1-\alpha)c_{t2}[-F'(x_{t2})]$$

$$(32) \quad c_{t2} = x_{t2} + (1+r)(x_{t1} - c_{t1}) - \lambda\theta - \lambda(1+r)t_1$$

Choosing the optimum values for x_{t2} and c_{t2} , we obtain

$$(33) \quad \Omega^{**}(x_{t1}, c_{t1}, g_1, t_1) \quad \text{with} \quad \Omega_{x_{t1}}^{**} = (1+r)\delta^{**} \quad \text{and} \quad \Omega_{c_{t1}}^{**} = -(1+r)\delta^{**}$$

where δ^{**} is the value of the multiplier at the optimum.

In order to see how the optimum values of x_{t2} and c_{t2} change with changes in the arguments of function Ω^{**} , we obtain from the first order conditions:

$$(34) \quad \begin{bmatrix} A_2 & 1 \\ -x_{t2} & c_{t2} \end{bmatrix} \begin{bmatrix} \hat{x}_{t2} \\ \hat{c}_{t2} \end{bmatrix} = \begin{bmatrix} -\beta_2(1+r)(\hat{t}_1 - \hat{g}_1) \\ (1+r)(x_{t1}\hat{x}_{t1} - c_{t1}\hat{c}_{t1}) - \lambda\theta(1+r)\hat{t}_1 \end{bmatrix}$$

The determinant of the system is

$$(35) \quad \Delta_2 = A_2 c_{t2} + x_{t2} > 0$$

and the changes in the values of the optimum choices for production and consumption of traded goods in the second period are

$$(36) \quad \hat{x}_{t2} = \Delta_2^{-1} [(1+r)(c_{t1}\hat{c}_{t1} - x_{t1}\hat{x}_{t1}) + c_{t2}\beta_2(1+r)\hat{g}_1 + (1+r)(\lambda\theta - \beta_2 c_{t2})\hat{t}_1]$$

$$(37) \quad \hat{c}_{t2} = \Delta_2^{-1} [A_2(1+r)(x_{t1}\hat{x}_{t1} - c_{t1}\hat{c}_{t1}) + x_{t2}\beta_2(1+r)\hat{g}_1 - (1+r)(\lambda\theta A_2 + x_{t2}\beta_2)\hat{t}_1]$$

From the first order condition (21), we also know that

$$(38) \quad \hat{\delta}^{**} = -\hat{c}_{t2}$$

Equations (33)-(38) show the private sector's choices for the second period consumption and production of traded goods under any arbitrary decision in period 1, and under the second alternative fiscal policy.

Maximization of utility as of period 1

Once we know which will be the private sector's choices in the second period under any arbitrary choice for the first period and under both alternative fiscal policies, we can determine the optimum choice for the first period. In the first period, the private

sector chooses x_{t1} and c_{t1} so as to maximize

$$(39) \quad \alpha \ln c_{t1} + (1-\alpha) \ln [F(x_{t1}) - (1-\lambda) q_1^{-1} g_1] + \\ + (1+\rho)^{-1} [p \Omega^*(x_{t1}, c_{t1}, g_1) + (1-p) \Omega^{**}(x_{t1}, c_{t1}, g_1, t_1)]$$

The first order conditions for a maximum are

$$(40) \quad \alpha c_{t1}^{-1} = (1+r)(1+\rho)^{-1} [p \delta^* + (1-p) \delta^{**}]$$

$$(41) \quad (1-\alpha) [-F'(x_{t1})] (F(x_{t1}) - (1-\lambda) [-F'(x_{t1})] g_1)^{-1} = \\ = (1+r)(1+\rho)^{-1} [p \delta^* + (1-p) \delta^{**}]$$

These two conditions determine the optimum choices for x_{t1} and c_{t1} . In order to see how these choices respond to changes in g_1 and t_1 , we differentiate (40) and (41), to get

$$(42) \quad -\hat{c}_{t1} = w^* \hat{\delta}^* + (1-w^*) \hat{\delta}^{**}$$

$$(43) \quad A_1 \hat{x}_{t1} + \beta_1 \hat{g}_1 = w^* \hat{\delta}^* + (1-w^*) \hat{\delta}^{**}$$

where

$$(44) \quad w^* = (p\delta^*)/[p\delta^* + (1-p)\delta^{**}]$$

and $\hat{\delta}^*$ and $\hat{\delta}^{**}$ are described by (29)-(30) and (37)-(38) respectively. Denoting by $\eta_{\delta^* z}$ and $\eta_{\delta^{**} z}$ the elasticities of δ^* and δ^{**} with respect to any variable z implied by (29)-(30) and (37)-(38) respectively, we can express the system (42)-(43) as

$$(45) \begin{bmatrix} -[w^* \eta_{\delta}^* x_{t1} + (1-w^*) \eta_{\delta}^{**} x_{t1}] & -[w^* \eta_{\delta}^* c_{t1} + (1-w^*) \eta_{\delta}^{**} c_{t1} + 1] \\ A_1 - [w^* \eta_{\delta}^* x_{t1} + (1-w^*) \eta_{\delta}^{**} x_{t1}] & -[w^* \eta_{\delta}^* c_{t1} + (1-w^*) \eta_{\delta}^{**} c_{t1}] \end{bmatrix} \begin{bmatrix} \hat{x}_{t1} \\ \hat{c}_{t1} \end{bmatrix} \\ - \begin{bmatrix} [(1-w^*) \eta_{\delta}^{**} t_1] \hat{t}_1 + [w^* \eta_{\delta}^* g_1 + (1-w^*) \eta_{\delta}^{**} g_1] \hat{g}_1 \\ [(1-w^*) \eta_{\delta}^{**} t_1] \hat{t}_1 + [w^* \eta_{\delta}^* g_1 + (1-w^*) \eta_{\delta}^{**} g_1 - \beta_1] \hat{g}_1 \end{bmatrix}$$

The determinant of the system is

$$(46) \quad \Delta_3 = A_1 + A_2(1+r)(A_1 c_{t1} + x_{t1}) [w^* \Delta_1^{-1} + (1-w^*) \Delta_2^{-1}] > 0 \\ = A_1 + A_2(1+r)(A_1 c_{t1} + x_{t1}) \Delta_1^{-1} > 0$$

where Δ_1 and Δ_2 are given by (27) and (3^c) respectively, and we use $\Delta_1 = \Delta_2$.

Effects of unsustainable policies

We proceed now to examine the effects of departures from sustainable policies in period 1 on production, consumption, the real exchange rate, and the current account. We start with the effects of changes in taxes.

$$(47) \quad \hat{x}_{t1} = \Delta_2^{-1} \Delta_3^{-1} (1-w^*) (1+r) (x_{t2} \beta_2 + \lambda \theta A_2) \hat{t}_1 > 0$$

$$(48) \quad \hat{c}_{t1} = - \Delta_2^{-1} \Delta_3^{-1} (1-w^*) A_1 (1+r) (x_{t2} \beta_2 + \lambda \theta A_2) \hat{t}_1 < 0$$

From (47), an increase in taxes in period 1 causes an increase in the production of traded goods and therefore a reduction in the production of nontraded goods. This shift in the production pattern is associated with a decline in the real exchange rate. From the equilibrium condition in the nontraded goods market (10), it follows that private sector's consumption of nontraded goods declines. From (48), private sector's consumption of traded goods also declines. The

current account in period 1 is equal to

$$(49) \quad CA = x_{t1} - c_{t1} - \lambda g_1$$

The change in the current account caused by an increase in taxes is then equal to

$$(50) \quad dCA = x_{t1}^{\wedge} - c_{t1}^{\wedge}$$

$$(51) \quad dCA = \Delta_2^{-1} \Delta_3^{-1} (1-w^*)(1+r) (x_{t2} \beta_2 + \lambda \theta A_2) (A_1 c_{t1} + x_{t1})^{\wedge} t_1 > 0$$

Thus, the increase in taxes improves the first period current account due to the increase in the production of traded goods and the decline in private sector's consumption of traded goods.

It is important to note that in all the expressions describing the effects of the reduction in taxes appears the factor $(1-w^*)$. This implies that if $w^*=1$ there is no effect at all in production, consumption, the real exchange rate, and the current account. From (44), $w^*=1$ implies $p=1$, which means that the increase in taxes in the present is certain to be compensated by a reduction in taxes in the future with the same present value. This is the well known Ricardian equivalence result.

Therefore, all the effects of an increase in taxes in the present come from the possibility that it will be used to finance higher public sector expenditure in the future, which would represent a real reduction in private sector lifetime consumption possibilities. This causes the private sector to reduce its consumption of both types of goods in the present. The reduction in the private sector's demand for nontraded goods causes a reduction in the production of those goods and an increase in the production of traded goods. All this is accompanied by a depreciation of the real exchange rate and an improvement in the current account.

We turn now to a discussion of the effects of an increase in public sector expenditure in the first period. In this case the results are less clear cut than the results of an increase in taxes. An increase in public sector expenditure has a direct effect on the demand for traded and nontraded goods in the present, in addition to

the effect of the endogenous change of the private sector's demand for those goods. Therefore we must be more specific about the assumptions needed to obtain unambiguous results. For the general case,

$$(52) \quad \hat{x}_{t1} = -\Delta_1^{-1} \Delta_3^{-1} \{ (1+r)(1-w^*)\beta_2 x_{t2} + \beta_1 \Delta_1 - [w^* \lambda (1-\alpha) - (1-\lambda)\alpha] (1+r) A_2 \theta (1-\alpha)^{-1} \} \hat{g}_1 > < 0$$

$$(53) \quad \hat{c}_{t1} = -\Delta_1^{-1} \Delta_3^{-1} (1+r) (A_2 \beta_1 x_{t1} + A_1 [w^* A_2 \lambda \theta - (1-w^*)\beta_2 x_{t2}]) \hat{g}_1 > < 0$$

Equation (52) indicates that the effect of an increase in public sector expenditure on the production of traded goods is ambiguous. However, it is sufficient to assume that the pattern of consumption is the same for the public and the private sector, $\alpha = \lambda$, for the production of traded goods to decline¹¹. We assume this to be the case. Associated with this change in the pattern of production there is an appreciation of the real exchange rate.

Equation (53) indicates that the change in private sector's consumption of traded goods is also ambiguous. Some restrictions on the shape of the transformation curve, or the assumption that $r = \rho$ would imply that the consumption of traded goods declines. If this is the case, using equations (40) and (42) it can be shown that private's sector consumption of nontraded goods also declines. The effect on the current account is given by

$$(54) \quad dCA = x_{t1} \hat{x}_{t1} - c_{t1} \hat{c}_{t1} - \lambda \hat{g}_1$$

In the general case, without any restriction in the parameters of the model,

$$(55) \quad dCA = -\Delta_1^{-1} \Delta_3^{-1} \{ (1-w^*)(1+r)(A_1 c_{t1} + x_{t1})(A_2 \lambda \theta + \beta_2 x_{t2}) + (A_2 c_{t2} + x_{t2})(A_1 \lambda \theta + \beta_1 x_{t1}) \} \hat{g}_1 < 0$$

¹¹

Actually, the weaker restriction $\lambda \leq \alpha$ is sufficient.

Therefore, an increase in public sector expenditure worsens the current account.

V. Overvalued and Undervalued Exchange Rates

Based on the results from the previous sections we can now link the concepts of "equilibrium" and misaligned real exchange rates to the notions of sustainable and unsustainable policies. The equilibrium real exchange rate can be defined as the real exchange rate that results from a set of sustainable policies. Misaligned exchange rates, on the other hand, would be associated with unsustainable policies. Thus, an overvalued exchange rate can be defined as the real exchange rate that results from unsustainable fiscal policies, such that if the present policies were to be maintained in the future the present value of lifetime public sector expenditure would be higher than the present value of taxes. An undervalued exchange rate can be defined as the result of fiscal policies that are unsustainable for the opposite reason.

These definitions are consistent with the usual associations between the type of misalignment, the level of the exchange rate and the current account of the balance of payments. For example, taking the case of a sustainable policy as the base or benchmark, an overvalued exchange rate would result from a reduction in taxes or an increase in public sector expenditure. According to our results, these changes will generally be associated with an appreciated exchange rate and a less favorable current account than in the benchmark case. An undervalued exchange rate would have the opposite implications.

From these definitions, misaligned exchange rates imply both, an intertemporal and an intersectoral shift in the economy's pattern of expenditure. Regarding the intertemporal shift, an overvalued exchange rate implies an increase in present expenditure, compensated by a reduction in future expenditure. This is reflected in the worsening of the present period current account of the balance of payments, and the corresponding improvement in the future current account. In contrast, an undervalued exchange rate implies a reduction in present